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Neutrino Masses and Mixing in Supersymmetric Models without R Parity

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We study neutrino masses and mixing in Supersymmetric Models without R parity and with generic soft Supersymmetry breaking terms. Neutrinos acquire mass from various sources: tree level neutrino–neutralino mixing, loop effects and non–renormalizable operators. Abelian horizontal symmetries (invoked to explain the smallness and hierarchy in quark parameters) replace R parity in suppressing neutrino masses. We find lower bounds on the mixing angles: $\sin\theta_{ij} \gtrsim m(\ell_i^-)/m(\ell_j^-)$ $(i<j)$ and unusual order of magnitude predictions for neutrino mass ratios: $m(\nu_e)/m(\nu_\mu) \sim \sin^2\theta_{12}$; $m(\nu_i)/m(\nu_\tau) \sim 10^{-7}\sin^2\theta_{i3}$ $(i=1,2)$. Bounds from laboratory experiments exclude $m_{\nu_\tau} \gtrsim 3\ MeV$ and cosmological constraints exclude $m_{\nu_\tau} \gtrsim 100\ eV$. Neither the solar nor the atmospheric neutrino problems are likely to be solved by $\nu_\mu - \nu_e$ oscillations. These conclusions can be evaded if holomorphy plays an important role in the lepton Yukawa couplings.

5/96

1. Introduction

The search for neutrino masses is one of the most promising directions to find evidence for the incompleteness of the Standard Model. Theoretical input is required in order to direct experiments to the most plausible values of neutrino masses and mixing angles. In particular, an understanding of the neutrino sector by the same means that explain the quark and charged lepton parameters would be desirable. Supersymmetry combined with horizontal symmetries can provide this understanding [1-6].

In Supersymmetric models with the MSSM particle content and with R parity (R_p), lepton number is violated by non–renormalizable terms only. Terms of the form $\frac{1}{M}L\phi_u L\phi_u$ (M is a high energy scale, L is the lepton doublet and ϕ_u is the $Y=+1/2$ Higgs doublet) lead to neutrino masses of the see–saw type, $m_\nu \sim \frac{\langle\phi_u\rangle^2}{M}$. The consequences of Abelian horizontal symmetries in this framework were investigated in ref. [3]. A number of interesting order of magnitude relations among the lepton parameters were found to hold in a large class of models:

$$\frac{m_{\nu_i}}{m_{\nu_j}} \sim \sin^2\theta_{ij}, \tag{1.1}$$

$$\sin\theta_{ij} \gtrsim \frac{m_{\ell_i}}{m_{\ell_j}}, \tag{1.2}$$

$$\frac{m_{\nu_i}}{m_{\nu_j}} \gtrsim \left(\frac{m_{\ell_i}}{m_{\ell_j}}\right)^2, \tag{1.3}$$

$$m_{\nu_e} \lesssim m_{\nu_\mu} \lesssim m_{\nu_\tau}, \tag{1.4}$$

where $i<j$ and ν_e,ν_μ,ν_τ denote the mass eigenstates with mixing of $\mathcal{O}(1)$ with e,μ,τ, respectively. Interestingly, predictions analogous to (1.2) and (1.4) apply to the quark sector (namely, $V_{ij} \gtrsim \frac{m(u_i)}{m(u_j)}, \frac{m(d_i)}{m(d_j)}$ and $V_{\rm CKM} \sim 1$) and are experimentally valid [3].

The same horizontal symmetries that explain the smallness and hierarchy in fermion parameters can naturally solve the problems related to lepton flavor and lepton number violation that arise in Supersymmetric models without R_p [7-9,5].[1] In this case, lepton

[1] While horizontal symmetries can rather easily take the role of R_p in suppressing lepton number violation, it is much more difficult to do so for baryon number violation [10]. Therefore, as in ref. [8], we simply assume that baryon number is a symmetry of Nature.

number is violated by renormalizable terms, and the resulting phenomenology is strikingly different from the one predicted by Supersymmetric R_p-symmetric models. In this work we examine the question of lepton masses and mixing angles in models of Supersymmetry without R_p but with a horizontal symmetry.

2. The Theoretical Framework

We work in the framework of the Abelian horizontal symmetry $\mathcal{H}$ that has been introduced in refs. [11-13]. $\mathcal{H}$ is explicitly broken by a small parameter λ to which we attribute charge –1 (and a numerical value of $\mathcal{O}(0.2)$, to explain the Cabibbo angle). This can be viewed as the effective low energy theory that comes from a Supersymmetric extension of the Froggatt–Nielsen mechanism at a high scale [14]. Then, the following selection rules apply:

(a) Terms in the superpotential that carry charge $n \geq 0$ under $\mathcal{H}$ are suppressed by $\mathcal{O}(\lambda^n)$, while those with $n < 0$ are forbidden by holomorphy;

(b) Terms in the Kähler potential that carry charge n under $\mathcal{H}$ are suppressed by $\mathcal{O}(\lambda^{|n|})$.

Without R_p (or lepton number), there is a–priori no distinction between the $Y = -1/2$ Higgs doublet ϕ_d and the three lepton doublets L_i. (Wherever convenient, we denote the four doublets by L_α, $\alpha = 0, 1, 2, 3$.) The four doublets, however, carry in general different horizontal charges H. We identify the Higgs doublet with the doublet field that carries the smallest (positive) charge, which we choose to be L_0 (we use interchangeably $L_0 \equiv \phi_d$), and we order the remaining doublets according to their charges:

$$H(L_1) \geq H(L_2) \geq H(L_3) \geq H(\phi_d) \geq 0\,. \tag{2.1}$$

A similar ordering is made for the three generations of $\bar{\ell}_i$ (charged lepton singlets), Q_i (quark doublets), and $\bar{d}_i$ (down quark singlets). The model is phenomenologically viable if (for $\tan\beta \sim 1$) the following condition holds [8]:

$$H(L_i) - H(\phi_d) \geq 3. \tag{2.2}$$

Our methods of analyzing lepton and neutralino mass matrices are described in detail in refs. [3] and [8], respectively. Specifically, we use the following selection rules to estimate

the magnitude of the various contributions. For the quadratic terms in the superpotential, $\mu_\alpha L_\alpha \phi_u$, it is :

$$\mu_\alpha \sim \begin{cases} \tilde\mu \lambda^{H(L_\alpha)+H(\phi_u)} & H(L_\alpha)+H(\phi_u) \geq 0, \\ \tilde m \lambda^{|H(L_\alpha)+H(\phi_u)|} & H(L_\alpha)+H(\phi_u) < 0. \end{cases} \tag{2.3}$$

Here $\tilde\mu$ is the natural scale for the μ terms and $\tilde m$ is the Supersymmetry breaking scale. For simplicity we assume that $\tilde\mu$ is $\mathcal{O}(\tilde m)$. As μ_0 is phenomenologically required to be of $\mathcal{O}(\tilde m)$, we take $H(\phi_d)+H(\phi_u) \sim 0$. Modifications to the case where the natural scale for μ is, say, $M_{\rm Planck}$ and it is suppressed down to $\tilde m$ by the horizontal symmetry, as in the models of [13] and [8], are straightforward. The selection rule for the coupling of the quadratic soft Supersymmetry breaking terms, $B_\alpha L_\alpha \phi_u$ (here L_α stand for the scalar components) is:

$$B_\alpha \sim \tilde m^2 \lambda^{|H(L_\alpha)+H(\phi_u)|}. \tag{2.4}$$

Finally, the selection rules for the trilinear terms $\lambda'_{\alpha jk} L_\alpha Q_j \bar d_k$ and $\lambda_{\alpha\beta k} L_\alpha L_\beta \bar\ell_k$ are

$$\lambda'_{\alpha jk} \sim \begin{cases} \lambda^{H(L_\alpha)+H(Q_j)+H(\bar d_k)} & H(L_\alpha)+H(Q_j)+H(\bar d_k) \geq 0, \\ 0 & H(L_\alpha)+H(Q_j)+H(\bar d_k) < 0, \end{cases} \tag{2.5}$$

$$\lambda_{\alpha\beta k} \sim \begin{cases} \lambda^{H(L_\alpha)+H(L_\beta)+H(\bar\ell_k)} & H(L_\alpha)+H(L_\beta)+H(\bar\ell_k) \geq 0, \\ 0 & H(L_\alpha)+H(L_\beta)+H(\bar\ell_k) < 0. \end{cases} \tag{2.6}$$

Note that λ'_{0jk} and λ_{0jk} are practically the Yukawa couplings for the down sector and for the charged lepton sector.

The order of magnitude relations (1.1)–(1.4) were derived in a large class of models where all entries in the lepton mass matrices carry positive charges. (They are actually applicable in a larger class of models, where the holomorphy–induced zero entries, if any, do not affect the physical parameters.) In this work we restrict ourselves to this class of models.

3. Neutrino Masses and Mixing

There are several important sources for neutrino masses in this framework, each giving a different scale: renormalizable tree–level mixing with neutralinos [15-24]; quark–squark and lepton–slepton loop corrections [16,25-32]; and non–renormalizable see–saw contributions [33-34]. We now discuss each contribution in turn. In our various estimates we take $\tan\beta \sim 1$.

(i) *Renormalizable tree–level contributions.*

These contributions arise when the μ–terms in the superpotential and the Supersymmetry breaking B terms in the scalar potential are misaligned, $B_\alpha \neq B\mu_\alpha$, or the Supersymmetry breaking scalar masses do not satisfy the eigenvalue condition $m^2_{\alpha\beta}\mu_\beta = \tilde{m}^2\mu_\alpha$ [8]. This yields misalignment between the VEVs $v_\alpha \equiv \langle L_\alpha \rangle$ and the μ_α terms,

$$\sin^2\xi = \frac{1}{2}\frac{\sum_{\alpha,\beta}(\mu_\alpha v_\beta - \mu_\beta v_\alpha)^2}{\mu^2 v_d^2}\,, \qquad (\mu^2 \equiv \mu_\alpha\mu_\alpha\,,\ v_d^2 \equiv v_\alpha v_\alpha) \tag{3.1}$$

which induces neutrino mixing with the neutralinos. Only one neutrino acquires a mass from this effect: $m_\nu \sim m_Z \sin^2\xi$ (here m_Z stands for the electroweak or Supersymmetry breaking scale). In the absence of any symmetry reason for alignment, we expect $\sin\xi \sim 1$ and the natural scale for m_ν is the electroweak scale. The further required suppression comes from $\mathcal{H}$-violation, $v_i/v_0 \sim \lambda^{[H(L_i)-H(\phi_d)]}$ [8]. Thus, (3.1) together with (2.1) gives

$$m_{\nu_\tau} \sim \lambda^{2[H(L_3)-H(\phi_d)]} m_Z. \tag{3.2}$$

The massive neutrino is then ν_τ , which is close to the interaction eigenstate with the smallest horizontal charge among the L_i. The experimental upper bound $m_{\nu_\tau} \leq 24\ MeV$ [35], when confronted with (3.2), is the source of the constraint (2.2).

(ii) *Quark–squark loop contributions.*

Loops with down quark and squarks contribute

$$\frac{1}{2}\,M^{\nu\ \mathrm{loop}}_{ij} \sim \frac{3\lambda'_{ikl}\lambda'_{jmn}}{16\pi^2}\frac{(M^d)_{lm}(\tilde{M}^{d2}_{LR})_{kn}}{\tilde{m}^2}, \tag{3.3}$$

where M^d is the d–quark mass matrix, and $\tilde{M}^{d2}_{LR}$ is the left–right sector in the $\tilde{d}$-squark mass-squared matrix. The experimental value of $V_{cb} \sim m_s/m_b$ strongly suggests that $H(\bar{d}_2) = H(\bar{d}_3)$ and consequently $(M^d)_{32} \sim (M^d)_{33} \sim m_b$ and $(\tilde{M}^{d2}_{LR})_{32} \sim (\tilde{M}^{d2}_{LR})_{33} \sim \tilde{m}m_b$. From this, together with (2.5), we learn that the largest contributions to (3.3) come from $k = m = 3$, and $l\,, n = 2, 3$. This, in general, gives mass to the two light neutrinos. Taking into account that $\lambda'_{033} \sim m_b/m_Z$, one obtains:

$$\begin{aligned} \frac{m^{\mathrm{loop}}_{\nu_\mu}}{m_{\nu_\tau}} &\sim \epsilon^{\mathrm{loop}}\lambda^{2[H(L_2)-H(L_3)]}, \\ \frac{m^{\mathrm{loop}}_{\nu_e}}{m_{\nu_\tau}} &\sim \epsilon^{\mathrm{loop}}\lambda^{2[H(L_1)-H(L_3)]}, \\ \epsilon^{\mathrm{loop}} &= \frac{3m_b^4}{8\pi^2 m_Z^4} \sim 10^{-7}, \end{aligned} \tag{3.4}$$

which implies the upper limit for the loop–induced masses $m_{\nu_i} \lesssim 10^{-7} m_{\nu_\tau} \lesssim 1\, eV$, $i = 1, 2$.

Note that if the Supersymmetry breaking trilinear scalar couplings are proportional to the Yukawa couplings ($\tilde{M}^{d2}_{LR} = \tilde{m} M^d$), the dominant quark–squark loop contributions to (3.3) yield a degeneracy in the mass matrix, and only one neutrino in (3.4) acquires mass. In the presence of a tree–level mass (3.2) for ν_τ , this mass eigenstate is ν_μ (and is close to the interaction eigenstate L_2). The same result applies also to the case that $(M^d)_{32} \ll (M^d)_{33}$ and $(\tilde{M}^{d2}_{LR})_{32} \ll (\tilde{M}^{d2}_{LR})_{33}$. Then a contribution to m_{ν_e} arises from quark-squark loops with e.g. $k{=}n{=}2$, $l{=}m{=}3$, giving $m_{\nu_e}/m_{\nu_\tau} \sim \frac{3 m_s^2 m_b^2}{8\pi^2 m_Z^4} \lambda^{2[H(L_1)-H(L_3)]}$, about two to three orders of magnitude below (3.4). This is somewhat smaller than the contribution from lepton–slepton loops discussed below.

(iii) *Lepton–slepton loop contributions.*

Loops with charged leptons and sleptons contribute

$$\frac{1}{2}\, M^{\nu\ \mathrm{loop}}_{ij} \sim \frac{\lambda_{ikl}\lambda_{jmn}}{16\pi^2} \frac{(M^\ell)_{lm} (\tilde{M}^{\ell 2}_{LR})_{kn}}{\tilde{m}^2}. \tag{3.5}$$

Using (2.6), we learn that the largest contribution from (3.5) has

$$\epsilon^{\mathrm{loop}} = \frac{m_\tau^4}{8\pi^2 m_Z^4} \sim 10^{-9}, \tag{3.6}$$

about two orders of magnitude lower than the dominant quark–squark contributions. As already mentioned, this contribution plays a significant role only when $\tilde{M}^{d2}_{LR} \simeq \tilde{m} M^d$ holds to a good approximation.

(iv) *Non–renormalizable contributions.*

The dimension–5 terms $\frac{1}{M} \nu_i \nu_j \phi_u \phi_u$ give [3]:

$$\frac{1}{2}\, M^{\nu\ \mathrm{nr}}_{ij} \sim \lambda^{H(L_i)+H(L_j)+2H(\phi_u)} \frac{m_Z^2}{M}. \tag{3.7}$$

This, in general, contributes to both light neutrinos:

$$\begin{aligned}
\frac{m^{\mathrm{nr}}_{\nu_\mu}}{m_{\nu_\tau}} &\sim \epsilon^{\mathrm{nr}} \lambda^{2[H(L_2)-H(L_3)]}, \\
\frac{m^{\mathrm{nr}}_{\nu_e}}{m_{\nu_\tau}} &\sim \epsilon^{\mathrm{nr}} \lambda^{2[H(L_1)-H(L_3)]}, \\
\epsilon^{\mathrm{nr}} &= \lambda^{2[H(\phi_u)+H(\phi_d)]} \frac{m_Z}{M} \sim 10^{-7} \left(\frac{10^9\ GeV}{M} \right).
\end{aligned} \tag{3.8}$$

The relative importance of the non–renormalizable and loop contributions to m_{ν_μ} and m_{ν_e} depends on the scale M (which is, roughly speaking, the natural scale for the masses of right–handed neutrinos). For $M \gtrsim 10^9\ GeV$, the leading contributions come from loops, while for $M \lesssim 10^9\ GeV$, the non–renormalizable contributions dominate.

Adding up the various contributions, and defining

$$\epsilon = \max(\epsilon^{\rm nr}, \epsilon^{\rm loop}), \tag{3.9}$$

leads to the following order of magnitude estimates for the neutrino masses and mixing angles:

$$\begin{aligned} m_{\nu_\tau}/m_Z &\sim \lambda^{2[H(L_3)-H(\phi_d)]}, \\ m_{\nu_i}/m_{\nu_\tau} &\sim \epsilon\lambda^{2[H(L_i)-H(L_3)]} \qquad (i=1,2), \end{aligned} \tag{3.10}$$

$$\sin\theta_{ij} \sim \lambda^{H(L_i)-H(L_j)} \qquad (i<j). \tag{3.11}$$

The charged lepton mass ratios are estimated to be

$$m_{\ell_i}/m_{\ell_j} \sim \lambda^{H(L_i)+H(\bar\ell_i)-H(L_j)-H(\bar\ell_j)}. \tag{3.12}$$

The charged current mixing matrix mixes not only the leptons among themselves, but also leptons with higgsinos and gauginos:

$$\begin{aligned} \sin\theta_{\nu_i\tilde\phi_d^-} &\sim \lambda^{H(L_i)-H(\phi_d)}, \\ \sin\theta_{\nu_i\tilde w^-} &\sim \lambda^{H(L_i)+H(\phi_u)}. \end{aligned} \tag{3.13}$$

The fact that the neutrino mass eigenstates have an isotriplet $\tilde w_3$ component in them, leads to flavor changing couplings of the Z-boson to neutrinos $\sim g\Omega_{ij}Z\nu_i\bar\nu_j$:

$$\Omega_{ij} \sim \lambda^{H(L_i)+H(L_j)+2H(\phi_u)}. \tag{3.14}$$

The estimates (3.10) and (3.11) give the following relations between neutrino masses and mixing angles:

$$\begin{aligned} \frac{m_{\nu_\mu}}{m_{\nu_\tau}} &\sim \epsilon\sin^2\theta_{23}, \\ \frac{m_{\nu_e}}{m_{\nu_\tau}} &\sim \epsilon\sin^2\theta_{13}, \\ \frac{m_{\nu_e}}{m_{\nu_\mu}} &\sim \sin^2\theta_{12}. \end{aligned} \tag{3.15}$$

There are two order of magnitude relations that are independent of ϵ :

$$\sqrt{\frac{m_{\nu_e}}{m_{\nu_\mu}}} \sim \sin\theta_{12} \sim \frac{\sin\theta_{13}}{\sin\theta_{23}}\,, \tag{3.16}$$

so that for the two light neutrinos (1.1) still holds. The order of magnitude inequality (1.2) is maintained:

$$\sin\theta_{ij} \gtrsim \frac{m_{\ell_i}}{m_{\ell_j}} \qquad (i<j). \tag{3.17}$$

The relation (1.4) is also maintained,

$$m_{\nu_e} \lesssim m_{\nu_\mu} \lesssim m_{\nu_\tau}. \tag{3.18}$$

However, unlike (1.3), the light neutrinos are much lighter than ν_τ:

$$\frac{m_{\nu_i}}{m_{\nu_\tau}} \lesssim \epsilon \qquad (i=1,2). \tag{3.19}$$

For a scale $M \gtrsim 10^9\ GeV$, this gives $\frac{m_{\nu_i}}{m_{\nu_\tau}} \lesssim 10^{-7}$. It is interesting that these models can naturally give mixing angles of $\mathcal{O}(1)$ with the third generation [24] while the corresponding mass ratios are very small. (For different mechanisms that give such a result, see [36-37].)

4. Theory Confronts Experiment

In this and the next sections we show that the order of magnitude relations derived in the last section, when combined with various experimental and cosmological constraints, exclude large regions of the mass–mixing parameter space. Most of our discussion in these two sections is independent of the question of R_p violation.

As the charged lepton masses are known, eq. (3.17) provides significant lower bounds on the lepton mixing angles. With $m_e/m_\mu \sim \lambda^3$ and $m_\mu/m_\tau \sim \lambda^2$, we get

$$\sin\theta_{23} \gtrsim \lambda^2, \quad \sin\theta_{13} \gtrsim \lambda^5, \quad \sin\theta_{12} \gtrsim \lambda^3. \tag{4.1}$$

The lower bound on $\sin\theta_{23}$ is particularly significant. First, if m_{ν_τ} is in the appropriate range, $\nu_\mu - \nu_\tau$ oscillations will be observed in the CHORUS, NOMAD and E803 experiments. Second, combining it with the upper bound $\sin^2\theta_{23} = BR(\pi \to \mu\nu_\tau) \leq 6\times 10^{-5}$ for $m_{\nu_\tau} \gtrsim 3\ MeV$ [38] results in

$$m_{\nu_\tau} \lesssim 3\ MeV. \tag{4.2}$$

Third, in combination with the bound on $\nu_\mu - \nu_\tau$ oscillations, $\sin^2 2\theta_{23} \leq 0.004$ for $\Delta m^2 \gtrsim 100\ eV^2$ [39], it gives[2]

$$\sin\theta_{23} \sim \lambda^2 \quad \text{for} \quad 10\ eV \lesssim m_{\nu_\tau} \lesssim 3\ MeV. \tag{4.3}$$

As we predict $\sin\theta_{13} \lesssim \sin\theta_{23}$, (4.3) implies also

$$\sin\theta_{13} \lesssim \lambda^2 \quad \text{for} \quad 10\ eV \lesssim m_{\nu_\tau} \lesssim 3\ MeV. \tag{4.4}$$

This bound is stronger than the bound from $\nu_e - \nu_\tau$ oscillations, $\sin^2 2\theta_{13} \leq 0.12$ for $\Delta m^2 \gtrsim 100\ eV^2$ [39], which, in this range, gives $\sin\theta_{13} \lesssim \lambda$. The latter bound, however, holds independently of whether holomorphy plays a role in determining the mixing angles.

Eqs. (4.2), (4.3) and (4.4) are applicable also in models with R_p because they result from (3.17) which holds independently of R_p violation.

5. Theory Confronts Cosmology

Cosmological considerations related to the age and the present energy density of the Universe provide a constraint on the mass and lifetime of neutrinos. For masses in the range 100 eV – a few MeV, the constraint reads (see e.g. [41])

$$m_{\nu_\tau}^2 \tau_{\nu_\tau} \lesssim 2 \times 10^{20}\ eV^2\ \text{sec}. \tag{5.1}$$

The framework of Abelian horizontal symmetries allows an estimate of the neutrino decay rates. It is interesting to find whether ν_τ could have a fast enough decay mode to fulfill (5.1) and have its mass above 100 eV.

The dominant decay modes are most likely those which proceed via gauge interactions. The bound (4.2) leaves only a very small window where the W–mediated tree level $\nu_\tau \to e^+e^-\nu_e$ is allowed. The rate can be estimated to be:

$$\begin{aligned} \frac{\Gamma(\nu_\tau \to e^+e^-\nu_e)}{\Gamma(\tau \to e\bar{\nu}_e\nu_\tau)} &= \frac{m_{\nu_\tau}^5}{m_\tau^5}\sin^2\theta_{13} \\ \Longrightarrow m_{\nu_\tau}^5 \tau_{\nu_\tau} &\sim \left(\frac{\lambda^5}{\sin\theta_{13}}\right)^2 3 \times 10^{41}\ eV^5\ \text{sec}. \end{aligned} \tag{5.2}$$

[2] The interpretation of oscillation experiments might change in the framework of Supersymmetry without R_p because new neutrino interactions are introduced [40]. In our case, however, the horizontal suppression makes these new interactions practically negligible.

Together with (4.2), we find that (5.1) is satisfied only for $\sin\theta_{13} \gtrsim \lambda^4$. Since there are charged particles in the final state, however, a stronger bound (from considerations of the cosmic microwave background radiation) applies, $\tau_{\nu_\tau} \lesssim 10^4$ sec. This cannot be satisfied for $m_{\nu_\tau} \lesssim 3\ MeV$ and $\sin\theta_{13} \lesssim \lambda^2$. Furthermore, detailed studies of the effects of a massive ν_τ during the Nucleosynthesis era [42-44] suggest that for $m_{\nu_\tau} \gtrsim 0.5\ MeV$, and independently of the decay modes, $\tau_{\nu_\tau} \lesssim 10^2$ sec is required, which closes the window even more firmly. Therefore we conclude that $\nu_\tau \to e^+e^-\nu_e$ does not open any window for a heavy ν_τ. Again, this conclusion holds also for models with R_p.

All other decay modes are flavor changing neutral current processes. There are three types of contributions to such processes:

(a) Loop diagrams with gauge particles, suppressed by the charged current mixing angles;

(b) Tree level Z–mediated decays suppressed by the Ω_{ij} mixing angles;

(c) Tree level slepton–mediated decays suppressed by the selection rules for the λ_{ijk} couplings.

The first class is common to models with and without R_p, but the other two are present only in R_p–violating models. In any case, we found that none of these channels is fast enough to allow $m_{\nu_\tau} \gtrsim 100\ eV$. For example, the rate for the Z–mediated $\nu_\tau \to 3\nu_\mu$ can be estimated to be:

$$\begin{aligned} \frac{\Gamma(\nu_\tau \to 3\nu_\mu)}{\Gamma(\tau \to e\bar{\nu}_e\nu_\tau)} &\sim \frac{m_{\nu_\tau}^5}{m_\tau^5}\Omega_{23}^2 \\ \Longrightarrow m_{\nu_\tau}^5 \tau_{\nu_\tau} &\sim \left(\frac{\lambda^6}{\Omega_{23}}\right)^2 10^{43}\ eV^5\ \text{sec}. \end{aligned} \tag{5.3}$$

This is significantly suppressed compared to (5.2) and does not satisfy (5.1).

As ν_τ is predicted to be the heaviest among the neutrinos, we conclude that in the framework of Supersymmetry and Abelian horizontal symmetry with or without R_p (and assuming that holomorphy does not play a role in determining $\sin\theta_{23}$)

$$m_{\nu_i} \lesssim 100\ eV \tag{5.4}$$

holds for all neutrino masses. We note that in the framework of a single $U(1)$ or Z_n broken by $\lambda \sim 0.2$, this requires $H(L_i) - H(\phi_d) \gtrsim 6$ which may be too large for reasonable models.

In some models of ref. [11], however, where the symmetry breaking parameters are much smaller, this can be achieved with charge differences ≤ 2.

6. Solar and Atmospheric Neutrinos

The upper bound $m_{\nu_\tau} \lesssim 100\ eV$ leads to even stronger bounds on m_{ν_μ} and m_{ν_e}. These bounds, however, depend on M and on $\tan\beta$. We believe that the most likely situation is (a) $M \gtrsim 10^9\ GeV$ (which, for example, applies in all models where the Supersymmetric extension of the Standard Model is valid up to some GUT scale) and (b) $\tan\beta \sim 1$ (which is the *natural* value [45-46]). Then in (3.9) $\epsilon \sim 10^{-7}$ leads to

$$m_{\nu_\tau} \lesssim 100\ eV, \quad m_{\nu_e} \lesssim m_{\nu_\mu} \lesssim 10^{-5}\ eV. \tag{6.1}$$

This bears important consequences for the solar and atmospheric neutrino problems. The value $m_{\nu_\mu} \lesssim 10^{-5}\ eV$ is inconsistent with

$$m^2_{\nu_\mu} - m^2_{\nu_e} \sim 6 \times 10^{-6}\ eV^2 \tag{6.2}$$

that is required to solve the solar neutrino problem through the MSW mechanism (see e.g. [47]), but is consistent with $m^2_{\nu_\mu} - m^2_{\nu_e} \sim 10^{-11}\ eV^2$ that could solve it through vacuum oscillations (see e.g. [48]). It is also inconsistent with

$$m^2_{\nu_\mu} - m^2_{\nu_e} \sim 10^{-2}\ eV^2 \tag{6.3}$$

that is required to solve the atmospheric neutrino problem through $\nu_\mu - \nu_e$ oscillations (see e.g. [49]).

The solar neutrino problem can still be solved by the MSW mechanism with $\nu_\tau - \nu_e$ oscillations (if $m_{\nu_\tau} \sim 10^{-3}\ eV$) or the atmospheric neutrino problem can be explained by $\nu_\tau - \nu_\mu$ oscillations (if $m_{\nu_\tau} \sim 10^{-1}\ eV$). However, we find that:

a. The two problems cannot be solved simultaneously;

b. The required horizontal charges are inconveniently large: $H(L_3) - H(\phi_d) \sim 10$ and $H(L_1) - H(\phi_d) \sim 12$ to get the small angle MSW solution for the solar neutrino problem, and $H(L_{2,3}) - H(\phi_d) \sim 8$ to solve the atmospheric neutrino problem;

c. Such a light ν_τ does not contribute to the dark matter and cannot play any role in structure formation. This would require

$$m_{\nu_\tau} \sim 10\ eV. \tag{6.4}$$

This situation is very different from the Supersymmetric models with R_p, where (6.4) and (6.2) can be simultaneously accommodated [3].

Things are different if we relax either of our two extra assumptions. As the tree level contribution to m_{ν_τ} is suppressed by $\tan^2\beta$ and the loop contributions are enhanced by $\tan^3\beta$, a large $\tan\beta$ would give a large ϵ (*e.g.* $\tan\beta \sim 20$ gives $\epsilon \sim 10^{-2}$). Then we can easily accommodate the dark matter (6.4) and solar neutrino (6.2) constraints ($H(L_2) = H(L_3) + 1 = H(\phi_d) + 6$). Alternatively, the solar and atmospheric neutrino problems can be solved simultaneously ($H(L_3) = H(L_2) = H(\phi_d) + 6$). It is non–trivial that, in this scenario, $H(L_2)$ and $H(L_3)$ which are fixed by the requirements on m_{ν_μ} and m_{ν_τ} give, at the same time, $\sin\theta_{23} = \mathcal{O}(1)$ as required to solve the atmospheric neutrino problem.

For $M < 10^9\ GeV$, the non–renormalizable contributions to m_{ν_μ} dominate over the loop corrections. In this case m_{ν_τ} can account for the dark matter, while m_{ν_μ} can accommodate the solar neutrino constraint (6.2) (however, this requires $M \lesssim 10^6\ GeV$).

Finally, if holomorphy does play an important role in the physical parameters, then even the conclusions of sections 3–5 can be evaded. For example, we can construct models where $\sin\theta_{23} \ll \lambda^2$, which would allow for m_{ν_τ} above the 3 MeV bound of section 4. Then, with $\sin\theta_{13} \sim \lambda^4$ (which is marginally compatible with the experimental limits [50-51]), the decay $\nu_\tau \to e^+e^-\nu_e$ can still open a window for m_{ν_τ} close to its experimental bound. Explicit examples of models where holomorphy induces approximate zeros in the mass matrices and affects physical parameters can be found in refs. [12,13,3].

7. Discussion

Models of Supersymmetry with Abelian horizontal symmetries have interesting implications for neutrino masses and mixing. We distinguish three cases:

(i) Models with R_p.

(ii) Models without R_p and with generic soft Supersymmetry breaking terms (up to the selection rules from the horizontal symmetry).

(iii) Models without R_p but with universal Supersymmetry breaking terms implying alignment at a high scale (namely $B_\alpha = B\mu_\alpha$ and $m^2_{\alpha\beta} = \tilde{m}^2\delta_{\alpha\beta}$).

Class (i) was analyzed in [3]. Class (ii) has been studied in this work. Class (iii), which yields a scenario quite different from the one investigated here, will be discussed in a forthcoming paper. We now compare the predictions of class (ii) with those of (i).

In a large class of models, where holomorphy does not introduce zeros in the mass matrices (or, if there are such zeros, they do not affect the order of magnitude of the physical parameters), we find the following order of magnitude relations among the mass ratios and mixing angles:

$$\begin{array}{cccc} & m_{\nu_\mu}/m_{\nu_\tau} & m_{\nu_e}/m_{\nu_\tau} & m_{\nu_e}/m_{\nu_\mu} \\ (i) & \sin^2\theta_{23} & \sin^2\theta_{13} & \sin^2\theta_{12} \\ (ii) & 10^{-7}\sin^2\theta_{23} & 10^{-7}\sin^2\theta_{13} & \sin^2\theta_{12} \end{array} \tag{7.1}$$

Note that the following relation among the mixing angles holds in both classes:

$$(i)\,,\ (ii)\ :\qquad \sin\theta_{13} \sim \sin\theta_{12}\,\sin\theta_{23}. \tag{7.2}$$

Furthermore, since $\sin\theta_{ij} \gtrsim m_{\ell_i}/m_{\ell_j}$ holds independently of R_p,

$$(i)\,,\ (ii)\ :\qquad \sin^2\theta_{23} \gtrsim 10^{-3},\quad \sin^2\theta_{13} \gtrsim 10^{-7},\quad \sin^2\theta_{12} \gtrsim 10^{-4}. \tag{7.3}$$

This leads to the following ranges for the mass ratios:

$$\begin{array}{cccc} & m_{\nu_\mu}/m_{\nu_\tau} & m_{\nu_e}/m_{\nu_\tau} & m_{\nu_e}/m_{\nu_\mu} \\ (i) & 10^{-3}\ -\ 1 & 10^{-7}\ -\ 1 & 10^{-4}\ -\ 1 \\ (ii) & 10^{-10}\ -\ 10^{-7} & 10^{-14}\ -\ 10^{-7} & 10^{-4}\ -\ 1 \end{array} \tag{7.4}$$

We conclude that measurements of the lepton mixing angles would test the Supersymmetric Abelian horizontal symmetry framework while measurements of neutrino mass ratios will serve to distinguish between models with or without R_p.

In ref. [41], it was shown that $m_{\nu_\mu}/m_{\nu_\tau} \gtrsim (m_\mu/m_\tau)^2$ together with cosmological considerations, strongly suggests that all neutrinos are lighter than $\mathcal{O}(100\ eV)$. Models without R_p predict $m_{\nu_\mu}/m_{\nu_\tau} \ll (m_\mu/m_\tau)^2$ but we still find that all neutrinos are lighter than $\mathcal{O}(100\ eV)$. This is a consequence of the fact that there is no decay mode large enough to fulfill the cosmological constraints on massive neutrinos.

In models with R_p, one can accommodate m_{ν_τ} to contribute sizeably to the cosmological dark matter, as well as m_{ν_μ} in the correct range required by the MSW solution of the solar neutrino problem. In models without R_p (and without any alignment condition) we find that, unless the scale of New Physics M is surprisingly low ($\lesssim 10^6\ GeV$), ν_μ is too light to play any role for matter enhanced oscillations of the solar ν_e's.

Finally, we emphasize that our various predictions are not entirely generic to models of Abelian horizontal symmetries. As described briefly in section 6, a large $\tan\beta$ and/or a small scale M would modify our discussion of the solar (and atmospheric) neutrino problem. But more important, one can construct models where holomorphy plays an important role and circumvents the otherwise model–independent predictions of eqs. (3.16), (3.17) and (3.18).

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